Dark Matter Direct Detection

Daeyeong Jeong
Department of Physics, Chungnam National University
jeong.physics@gmail.com
(Dated: November 8, 2025)

It is my honor to give a presentation at the first JCH HEP School. Whereas the previous lecture addressed the overview of dark matter and its indirect detection, this lecture aims to provide basic knowledge on direct dark matter detection. In addition, we will briefly introduce Light Dark Matter, which has been a subject of active research in direct detection, and discuss how the features of direct detection depend on the dark matter mass.

I. DARK MATTER DIRECT DETECTION: DARK MATTER WIND

In the standard picture of cold, non-relativistic dark matter (DM) in the Milky Way halo, the phase-space distribution is approximately stationary in the Galactic frame. Consequently, as the Earth moves through the halo, a relative velocity develops between the Earth and the DM population, giving rise to a "DM wind" whose mean direction is opposite to the Earth's motion. We decompose the Earth's total velocity as

$$\mathbf{v}_{\text{tot}} = \mathbf{v}_{\text{c}} + \mathbf{v}_{\text{s}} + \mathbf{v}_{\text{e,rev}} + \mathbf{v}_{\text{e,rot}} \tag{1}$$

where $\mathbf{v}_{\text{c}}$ denotes the Galactic rotation velocity at the solar radius, $\mathbf{v}_{\text{s}}$ is the Sun's peculiar velocity with respect to the local standard of rest, $\mathbf{v}_{\text{e,rev}}$ is the Earth's orbital (revolution) velocity about the Sun, and $\mathbf{v}_{\text{e,rot}}$ is the Earth's rotational velocity about its spin axis. The mean DM-wind velocity in the Galactic frame is therefore

$$\mathbf{v}_{\text{DM}} = -\mathbf{v}_{\text{tot}} \tag{2}$$

A. Galactic Rotation $\mathbf{v}_{\text{c}}$

$$\mathbf{v}_{\text{c}} = \text{v}_{\text{c}}\hat{y}_{\text{g}} \tag{3}$$

Here, $\hat{y}_{\text{g}}$ denotes the direction of Galactic rotation in the Galactic frame, and v_{c} is the local circular speed, approximately 220 km/s with 10% statistical error.

B. Sun's Peculiar Velocity $\mathbf{v}_{\text{s}}$

The Sun's peculiar velocity in the LSR is given by

$$\mathbf{v}_s = U\hat{\mathbf{x}}_g + V\hat{\mathbf{y}}_g + W\hat{\mathbf{z}}_g \tag{4}$$

and

$$(U, V, W)_\odot = \left(11.1^{+0.69}_{-0.75},\ 12.24^{+0.47}_{-0.47},\ 7.25^{+0.37}_{-0.36}\right)\ \text{km/s} \tag{5}$$

C. Earth Revolution $\mathbf{v}_{\text{e,rev}}$

The Earth revolves around the Sun on an elliptical orbit, and the orbital plane undergoes precession. Therefore, both the time-dependent Earth–Sun geometry and the orbital corrections must be taken into account. The Earth's orbital velocity is given by

$$\begin{aligned}\mathbf{v}_{e,\text{rev}} = v_\oplus \Big(&\cos\beta_x \left[\sin(L-\lambda_x) + e\sin(L+g-\lambda_x)\right]\hat{\mathbf{x}}_g \\ &+ \cos\beta_y \left[\sin(L-\lambda_y) - e\sin(L+g-\lambda_y)\right]\hat{\mathbf{y}}_g \\ &+ \cos\beta_z \left[\sin(L-\lambda_z) - e\sin(L+g-\lambda_z)\right]\hat{\mathbf{z}}_g \Big)\end{aligned} \tag{6}$$

Here, $v_\oplus = 29.79$ km/s denotes the Earth's revolution speed, e = 0.0167023 is the orbital ellipticity, and ($\beta_{x,y,z}$, $\lambda_{x,y,z}$) represent the ecliptic latitude and longitude. The mean longitude L and mean anomaly g are given by

$$\begin{aligned} L &= 279^\circ.344 + 0.9856474\,d \\ g &= 356^\circ.154 + 0.9856003\,d \end{aligned} \tag{7}$$

The quantity d is the fractional day number measured from 0 UT on December 31, 2014, defined as

$$d = [365.25\,\tilde{Y}] + [30.61\,(\tilde{M}+1)] + D + \frac{\text{UT}}{24} - 736041 \tag{8}$$

Here, $[\cdot]$ denotes the floor operation. For $M \leq 2$, we take $\tilde{Y} = Y - 1$ and $\tilde{M} = M + 12$, while for $M > 2$, we use $\tilde{Y} = Y$ and $\tilde{M} = M$.

The ecliptic latitude and longitude are obtained from

$$\begin{aligned} (\beta_x, \lambda_x) &= (5^\circ.538, 267^\circ.050) + (0^\circ.013, 1^\circ.397)\,T \\ (\beta_y, \lambda_y) &= (-59^\circ.574, 347^\circ.546) + (0^\circ.002, 1^\circ.375)\,T \\ (\beta_z, \lambda_z) &= (29^\circ.811, 180^\circ.234) + (0^\circ.001, 1^\circ.404)\,T \end{aligned} \tag{9}$$

Finally, the parameter T = d/36525 specifies the number of Julian centuries, which is used to correct for the precession of the equinox.

D. Earth Rotation $\mathbf{v}_{\mathrm{e,rot}}$

The Earth's rotational velocity is given by

$$\mathbf{v}_{e,\mathrm{rot}} = -v_{e,\mathrm{rot}}^{\mathrm{eq}} \cos\lambda_{\mathrm{lab}} \hat{\mathcal{W}} \tag{10}$$

Here, $\mathrm{v}_{\mathrm{e,rot}}^{\mathrm{eq}}$ denotes the rotational speed at the Earth's equator, λ_{lab} is the latitude of the laboratory, and $\hat{\mathcal{W}}$ represents the westward direction in the laboratory frame.

The transformation from the laboratory frame to the Galactic frame is expressed as

$$\mathbf{v}_{e,\mathrm{rot}} = -v_{e,\mathrm{rot}}^{\mathrm{eq}} \cos\lambda_{\mathrm{lab}} \Big[(a_x \sin t_{\mathrm{lab}}^0 - a_y \cos t_{\mathrm{lab}}^0)\, \hat{\mathbf{x}}_g + (b_x \sin t_{\mathrm{lab}}^0 - b_y \cos t_{\mathrm{lab}}^0)\, \hat{\mathbf{y}}_g + (c_x \sin t_{\mathrm{lab}}^0 - c_y \cos t_{\mathrm{lab}}^0)\, \hat{\mathbf{z}}_g \Big] \tag{11}$$

where a_i, b_i, c_i are elements of transformation matrix from Galactic frame to the equatorial frame.

Here, $t_{\mathrm{lab}}^0 = \mathrm{GMST} + \mathrm{longitude}$ denotes the local sidereal time, incorporating both the longitude of the laboratory and the time of observation.

E. Dark Matter Wind Velocity

From Eqs. (1), (3), (4), (6), and (11), the Earth's velocity in the Galactic reference frame, $\mathbf{v}_{\mathrm{tot}}$, is given by

$$\begin{aligned}
\mathbf{v}_{\mathrm{tot}} = & \Big(U + v_\oplus \cos\beta_x \left[\sin(L-\lambda_x) + e\sin(L+g-\lambda_x)\right] - v_{e,\mathrm{rot}}^{\mathrm{eq}} \cos\lambda_{\mathrm{lab}} \left(a_x \sin t_{\mathrm{lab}}^0 - a_y \cos t_{\mathrm{lab}}^0\right) \Big) \hat{\mathbf{x}}_g \\
& + \Big(v_{\mathrm{c}} + V + v_\oplus \cos\beta_y \left[\sin(L-\lambda_y) - e\sin(L+g-\lambda_y)\right] - v_{e,\mathrm{rot}}^{\mathrm{eq}} \cos\lambda_{\mathrm{lab}} \left(b_x \sin t_{\mathrm{lab}}^0 - b_y \cos t_{\mathrm{lab}}^0\right) \Big) \hat{\mathbf{y}}_g \\
& + \Big(W + v_\oplus \cos\beta_z \left[\sin(L-\lambda_z) - e\sin(L+g-\lambda_z)\right] - v_{e,\mathrm{rot}}^{\mathrm{eq}} \cos\lambda_{\mathrm{lab}} \left(c_x \sin t_{\mathrm{lab}}^0 - c_y \cos t_{\mathrm{lab}}^0\right) \Big) \hat{\mathbf{z}}_g
\end{aligned} \tag{12}$$

According to Eq. (2), the dark matter velocity in the Galactic frame is then expressed as

$$\begin{aligned}
\mathbf{v}_{\mathrm{DM}} = & - \Big(U + v_\oplus \cos\beta_x \left[\sin(L-\lambda_x) + e\sin(L+g-\lambda_x)\right] - v_{e,\mathrm{rot}}^{\mathrm{eq}} \cos\lambda_{\mathrm{lab}} (a_x \sin t_{\mathrm{lab}}^0 - a_y \cos t_{\mathrm{lab}}^0) \Big) \hat{\mathbf{x}}_g \\
& - \Big(\mathrm{v_c} + V + v_\oplus \cos\beta_y \left[\sin(L-\lambda_y) - e\sin(L+g-\lambda_y)\right] - v_{e,\mathrm{rot}}^{\mathrm{eq}} \cos\lambda_{\mathrm{lab}} (b_x \sin t_{\mathrm{lab}}^0 - b_y \cos t_{\mathrm{lab}}^0) \Big) \hat{\mathbf{y}}_g \\
& - \Big(W + v_\oplus \cos\beta_z \left[\sin(L-\lambda_z) - e\sin(L+g-\lambda_z)\right] - v_{e,\mathrm{rot}}^{\mathrm{eq}} \cos\lambda_{\mathrm{lab}} (c_x \sin t_{\mathrm{lab}}^0 - c_y \cos t_{\mathrm{lab}}^0) \Big) \hat{\mathbf{z}}_g
\end{aligned} \tag{13}$$

Since direct detection experiments are conducted on the Earth, the Galactic-frame velocity must be transformed into the laboratory frame to obtain the DM velocity at a specific location and time. For the computational procedure of this transformation, see Ref [1]. Through this procedure, the DM wind acquires an annual modulation due to the Earth's revolution, reaching a speed of about 250 km/s in June and about 220 km/s in December. In addition, the Earth's rotation induces a daily modulation of order 0.5 km/s, whose amplitude depends on the latitude of the laboratory.

F. Dark Matter Velocity Distribution $f(\mathbf{v})$

We have obtained the DM wind velocity. However, in practice the DM has a truncated Maxwellian velocity distribution in the lab frame, cut off at the escape velocity with respect to the laboratory-frame DM speed v, given by

$$f_{\mathrm{lab}}(\mathbf{v}, t) = \frac{1}{N(v_0)} \exp\left[-\frac{|\mathbf{v} - \mathbf{v}_{\mathrm{wind}}(t)|^2}{v_0^2} \right] \Theta(v_{\mathrm{esc}} - |\mathbf{v} - \mathbf{v}_{\mathrm{wind}}(t)|)\,. \tag{14}$$

The escape velocity $v_{\rm esc}$ is taken to be 500–600 km/s, and the normalization factor $N(v_0)$ is

$$N(v_0) = \pi^{3/2} v_0^3 \left[\mathrm{erf}\left(\frac{v_{\rm esc}}{v_0}\right) - \frac{2}{\sqrt{\pi}} \frac{v_{\rm esc}}{v_0} \exp\left(-\frac{v_{\rm esc}^2}{v_0^2}\right) \right] \tag{15}$$

II. WIMPS DIRECT DETECTION : NUCLEAR SCATTERING

Direct detection experiments aim to observe the interaction of Galactic dark matter particles with target materials in laboratory. The DM wind causes dark matter particles to scatter off nuclear in the detector with an extremely low probability, producing measurable recoil signals.

First, we consider direct detection searches for weakly interacting massive particles (WIMPs), dark matter candidates with masses above a few GeV.

A. Kinematics of Nuclear Recoil

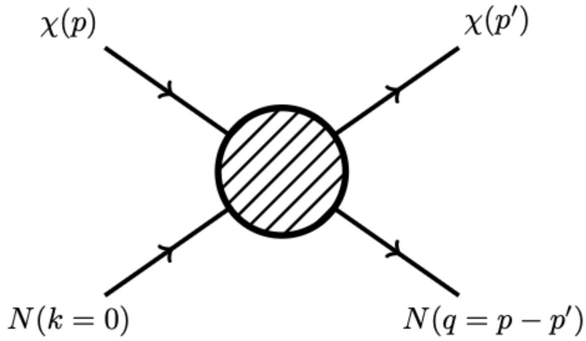

FIG. 1. Scattering of DM on nucleus N.

In the case of elastic scattering between a dark matter particle and a nucleus, as illustrated in Fig. 1, the initial energy is given by

$$E_i = \frac{p^2}{2m_\chi} \quad \mathbf{p} = m_\chi \mathbf{v} \tag{16}$$

If the momentum transfer is denoted by $\mathbf{q} = p - p'$, the final energy becomes

$$E_f = \frac{(\mathbf{p}-\mathbf{q})^2}{2m_\chi} + \frac{\mathbf{q}^2}{2m_N} \tag{17}$$

Defining $\cos\theta = \hat{\mathbf{p}} \cdot \hat{\mathbf{q}}$, the conservation of energy leads to the relation

$$\frac{|\mathbf{p}|\,|\mathbf{q}|\cos\theta}{m_\chi} = \frac{\mathbf{q}^2}{2\mu_{\chi N}} \tag{18}$$

where $\mu_{\chi N} = \frac{m_\chi m_N}{m_\chi + m_N}$ is the reduced mass of the DM–nuclear system.

The maximum momentum transfer is therefore

$$|\mathbf{q}|_{\rm max} = \frac{2\,\mu_{\chi N}\,|\mathbf{p}|}{m_\chi} = 2\,\mu_{\chi N}\,v \tag{19}$$

where $v = \sim 10^{-3}$ represents the typical DM velocity. For WIMP–nuclear scattering, one finds $\mu_{\chi N} \sim$ 10–100 GeV and $|\mathbf{q}|_{\rm max} \simeq$ 20–200 MeV. This corresponds to a characteristic length scale of $1/|\mathbf{q}| \sim$ 1–10 fm, comparable to the nuclear radius ($\sim$ 5 fm), indicating that the scattering is coherent.

The corresponding maximum nuclear recoil energy is given by

$$E_R^{\rm max} = \frac{|\mathbf{q}|^2_{\rm max}}{2m_N} = \frac{2\mu\chi N^2 v^2}{m_N} \simeq 20\text{–}200 \text{ keV} \tag{20}$$

In what follows, we derive the differential cross section for DM–nucleus scattering and, by integrating it over the dark matter velocity distribution, obtain the differential recoil rate.

B. Calculate Matrix Elements : Mapping the interaction high scale to low scale

As discussed above, for spin-independent interactions mediated by heavy particles such as the Z or Higgs boson, WIMP scattering is coherent over the whole nucleus. Although these mediators lie at the electroweak scale, $m_{\rm med} \sim 10^2$ GeV, the typical nuclear recoil energy from Eq.(20) is only $E_R^{\rm max} \sim 10^{-5}$–$10^{-4}$GeV, i.e. lower by about six to seven orders of magnitude. Since $q^2 \ll m_{\rm med}^2$ in this regime, the mediator can be integrated out and the interaction is well described by a contact effective field theory. One first matches the heavy-mediator theory onto DM–quark operators at the high scale and then runs and matches these onto DM–nucleon operators at the hadronic scale, which form the starting point for the nuclear response and direct-detection observables.

Let us consider the interaction of fermionic DM mediated by weak-scale particles, i.e. the WIMP case. Below the weak scale, $\sim m_Z$, the DM–quark interaction, i.e. the 'effective' Lagrangian, can be written in terms of the following dimension-6 operators as

$$\mathcal{L}_{\rm eff} = \frac{C_V}{m_Z^2}\,(\bar\chi\gamma^\mu\chi)(\bar q\gamma_\mu q) \;+\; \frac{C_A}{m_Z^2}\,(\bar\chi\gamma^\mu\gamma^5\chi)(\bar q\gamma_\mu\gamma^5 q) \;+\; \frac{C_S m_q}{m_h^2}\,(\bar\chi\chi)(\bar q q) \;+\; \dots \tag{21}$$

Here $\mathrm{C_i}$ are Wilson coefficients determined by the underlying couplings, and v denotes the Higgs vacuum expectation value.

The first term arises from the vector coupling of the Z boson in DM–quark scattering. Since γ^μ is the Lorentz vector current, $\bar\chi\gamma^\mu\chi$ and $\bar q\gamma_\mu q$ represent the DM and quark vector currents, respectively; this leads to a spin-independent interaction. The factor $\frac{1}{m_{z^2}}$ reflects the mediator-mass suppression. This is the interaction most commonly probed in direct-detection experiments.

The second term is induced by the axial-vector component of the Z boson. Because γ^5 selects the spin (more precisely, the chirality) structure, the operator

$$\frac{C_A}{m_Z^2}(\bar\chi\gamma^\mu\gamma^5\chi)(\bar q\gamma_\mu\gamma^5 q) \tag{22}$$

corresponds to a spin-dependent interaction. Such couplings are relevant for target nuclei with nonzero spin (e.g. $^{19}\mathrm{F}, {}^{129}\mathrm{Xe}$).

The third term corresponds to a scalar coupling mediated by Higgs exchange.

$$\frac{C_S m_q}{m_h^2}\,(\bar\chi\chi)(\bar q q) \tag{23}$$

Where $\bar\chi\chi$ and $\bar q q$ are the DM and quark scalar densities. This interaction is spin-independent and becomes important in models where Higgs exchange dominates.

We now run Eq. (21) down to the QCD scale. At this stage, loop corrections induce operator mixing, so the Wilson coefficients $\mathrm{C_i}$ can be modified. For example, suppose that at the high scale we have a scalar operator of the form $(\bar\chi\chi)(\bar q q)$ as in Eq. (21). In the actual DM–quark interaction, heavy quarks such as c,b,t do not couple to DM at tree level. However, through QCD loop effects a gluon can mediate the interaction between DM and these heavy quarks, which in turn generates an operator of the form $(\bar\chi\chi)\,G^a_{\mu\nu}G^{a\mu\nu}$.. As a result, the Wilson coefficients $\mathrm{C_i}$ are shifted by the loop-induced mixing. In this lecture, we take $\mathrm{C_i}$ to be the values at the QCD scale.

Next step is matching the quark-level operators onto nucleon operators. This is done using nucleon matrix elements of the partonic operators; see Refs [2–5] for details.

1. *Spin Independent Scattering*

For the vector current of light quarks (u,d,s), we can write

$$\langle n(k') \mid \bar q\gamma^\mu q \mid n(k)\rangle = \bar u_n(k')\left[F_1^{q,n}(q^2)\,\gamma^\mu + \frac{i}{2m_n}F_2^{q,n}(q^2)\,\sigma^{\mu\nu}q_\nu\right]u_n(k) \tag{24}$$

where $q = p - p' = k' - k$ is the momentum transfer, n denotes a nucleon (proton p or neutron n), and we neglect isospin-breaking differences in the couplings. Since dark matter is cold, we may take $q^2 \to 0$. In this limit, the form factor $\mathrm{F_1^{q,n}}(0)$ measures the quark content inside the nucleon.

$$F_1^{u,p}(0) = 2 \qquad F_1^{d,p}(0) = 1 \qquad F_1^{s,p}(0) = 0 \tag{25}$$

The second form factor $F_2^{q,n}(0)$ encodes the contribution of the quark flavor to the anomalous magnetic moment of the nucleon, but in direct detection one has $q^\mu/m_n \ll 1$, so the second term can be neglected.

We now apply the non-relativistic limit to the nucleon operators. Since we want to approximate the Lorentz-invariant expression in Eq. (24) by a non-relativistic one, we must work with Galilean-invariant operators rather than Lorentz-covariant ones. In this case, the set of allowed operators reduces to

$$1 \quad \text{(scalar)} \tag{26}$$

$$\left.\begin{aligned} \mathbf{q} &\equiv \mathbf{p} - \mathbf{p}' = \mathbf{k}' - \mathbf{k}, \\ \mathbf{v}_{\rm rel}^{\perp} &\equiv \mathbf{v}_{\rm rel} - \frac{\mathbf{q}}{2\mu_{\chi n}} \end{aligned}\right\} \quad \text{(3-vectors)}$$

$$\left.\begin{aligned} &\mathbf{S}_\chi \\ &\mathbf{S}_n \end{aligned}\right\} \quad \text{(pseudovectors)}$$

where $\mathbf{v}_{\rm rel} \equiv \mathbf{v}_{\chi,i} - \mathbf{v}_{n,i}$, and satisfies $\mathbf{v}_{\rm rel}^{\perp} \cdot \mathbf{q} = 0$. $\mathbf{v}_{\chi,i}, \mathbf{v}_{n,i}$ are the initial velocities of the DM and the nucleon, respectively.

To apply this to Eq. (24), we separate $\bar{u}_n(k')\gamma^\nu u_n(k)$ into its scalar part $\bar{u}_n(k')\gamma^0 u_n(k)$ and 3-vector part $\bar{u}_n(k')\gamma^i u_n(k)$.

The Dirac spinor can be written as

$$u(k) = \sqrt{E_k + m} \begin{pmatrix} \xi_s \\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{k}}{E_k + m}\,\xi_s \end{pmatrix} \tag{27}$$

where ξ_s is a two-component Pauli spinor and $E_k = \sqrt{m^2 + \mathbf{k}^2} \simeq m + \mathbf{k}^2/(2m)$. In the nonrelativistic limit ($|\mathbf{k}|/m \ll 1$), we have $E_k + m \simeq 2m$, so

$$u(k) \simeq \sqrt{2m} \begin{pmatrix} \xi_s \\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{k}}{2m}\,\xi_s \end{pmatrix} \tag{28}$$

$$\bar{u}(k') = u(k')^\dagger \gamma^0 = \left(\sqrt{2m}\,\xi_{s'}^\dagger \;,\; \sqrt{2m}\,\frac{\xi_{s'}^\dagger\,\boldsymbol{\sigma}\cdot\mathbf{k}'}{2m} \right) \tag{29}$$

First consider the scalar part. Using

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \tag{30}$$

we obtain

$$\bar{u}(k')\gamma^0 u(k) = u(k')^\dagger u(k)$$
$$\simeq \left(\sqrt{2m}\,\xi_{s'}^\dagger \;,\; \sqrt{2m}\,\frac{\xi_{s'}^\dagger\,\boldsymbol{\sigma}\cdot\mathbf{k}'}{2m} \right) \begin{pmatrix} \sqrt{2m}\,\xi_s \\ \sqrt{2m}\,\dfrac{\boldsymbol{\sigma}\cdot\mathbf{k}}{2m}\,\xi_s \end{pmatrix} \tag{31}$$

In the nonrelativistic limit $|\mathbf{k}|/m_n$, $|\mathbf{k}'|/m_n \ll 1$,

$$\bar{u}_n(k')\gamma^0 u_n(k) = 2m_n\,\xi_{s'}^\dagger \xi_s . \tag{32}$$

Next, for the 3-vector component we use

$$\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \tag{33}$$

and find

$$\bar{u}(k')\gamma^i u(k) \tag{34}$$
$$= \left(\sqrt{2m}\,\xi_{s'}^\dagger \;,\; \sqrt{2m}\,\frac{\xi_{s'}^\dagger\,\boldsymbol{\sigma}\cdot\mathbf{k}'}{2m} \right) \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \begin{pmatrix} \sqrt{2m}\,\xi_s \\ \sqrt{2m}\,\dfrac{\boldsymbol{\sigma}\cdot\mathbf{k}}{2m}\,\xi_s \end{pmatrix}$$

Using the Pauli identity $\sigma^i\sigma^j = \delta^{ij} + i\epsilon^{ijk}\sigma^k$, this becomes

$$\bar{u}(k')\gamma^i u(k) = \xi_{s'}^\dagger \left[(k' + k)^i - i\epsilon^{ijk} q^j \sigma^k \right] \xi_s \tag{35}$$

	Axial charges Δ_q^n		
Nucleon	Δ_u^n	Δ_d^n	Δ_s^n
Neutron	$-0.46(4)$	$0.80(3)$	$-0.12(8)$
Proton	$0.80(3)$	$-0.46(4)$	$-0.12(8)$

TABLE I. Axial charges for nucleons. Values are shown for both neutron and proton states.

In the nonrelativistic limit $|\mathbf{k}|/m_n$, $|\mathbf{k}'|/m_n \ll 1$, these spatial terms are suppressed and can be neglected.

Thus

$$\bar{u}_n(k')\gamma^\nu u_n(k) \simeq \bar{u}_n(k')\gamma^0 u_n(k) = 2m_n\,\xi_{s'}^\dagger \xi_s \tag{36}$$

Applying the same reduction to $\bar{\chi}\gamma^\mu\chi$, the matrix element for vector exchange takes the form

$$\mathcal{M} = \frac{12\,C_V}{m_Z^2}\, m_n m_\chi\, (\xi_{t'}^\dagger \xi_t)\, (\xi_{s'}^\dagger \xi_s) \tag{37}$$

i.e. it factorizes into DM and nucleon spinors. The operator $\bar{\chi}\chi\,\bar{q}q$ yields the same structure up to an overall coefficient. Such scattering is referred to as spin-independent scattering.

2. *Spin Dependent Scattering*

The vector current for light quarks (u, d, s) can likewise be written as

$$\langle n(k') \mid \bar{q}\gamma^\mu\gamma^5 q \mid n(k)\rangle \tag{38}$$
$$= \bar{u}_n(k') \left[F_A^{q,n}(q^2)\,\gamma^\mu\gamma^5 + \frac{1}{2m_n} F_{P'}^{q,n}(q^2)\, q^\mu\gamma^5 \right] u_n(k)$$

The $q^2 \to 0$ limits of these form factors are determined from a combination of semileptonic decays, hadronic scattering data, and lattice-QCD results. The axial charges $F_A^{q,n}(0) \equiv \Delta_q^n$ are summarized in Table I. This pseudovector current is the main source of spin-dependent scattering.

As in the spin-independent case, we keep only the leading term, $\Delta_q^n\, \bar{u}_n(k')\gamma^\mu\gamma^5 u_n(k)$. This can be decomposed into the pseudo-scalar part $\bar{u}_n(k')\gamma^0\gamma^5 u_n(k)$ and the pseudo-vector part $\bar{u}_n(k')\gamma^i\gamma^5 u_n(k)$. The pseudo-scalar piece is proportional to $\mathbf{q}\cdot\mathbf{S}_n$ and vanishes in the non-relativistic limit, so we retain only the pseudo-vector part. The matrix element can then be written as

$$\mathcal{M} = \sum_q \frac{16\,C_A\, m_\chi m_n}{m_Z^2}\, \Delta_q\, \xi_{t'}^\dagger \mathbf{S}_\chi \xi_t \cdot \xi_{s'}^\dagger \mathbf{S}_n \xi_s \tag{39}$$

For spin-$\frac{1}{2}$ fermions this may be expressed in terms of Pauli matrices as

$$\mathcal{M} = \sum_q \frac{4\,C_A\, m_\chi m_n}{m_Z^2}\, \Delta_q\, (\zeta^\dagger \sigma^i \zeta)\, (\xi^\dagger \sigma^i \xi) \tag{40}$$

This interaction is referred to as spin-dependent scattering and is relevant for nuclei with nonzero spin, such as ^{129}Xe ($J = \frac{1}{2}$) or ^{133}Cs ($J = \frac{7}{2}$). Its overall size is typically much smaller than that of spin-independent scattering.

C. Calculate Matrix Elements : Calculate the Scattering Rate

We now apply the DM–nucleon scattering matrix elements obtained above to a specific nuclear target to compute the scattering rate.

Define the nuclear form factors as follows:

$$|\langle N|\mathbf{S}_n(q)|N\rangle|^2 = F^2_{\rm spin}(q) \tag{41}$$

$$|\langle N|\bar{n}n|N\rangle|^2 = F^2_{\rm mass}(q) \tag{42}$$

To compute these form factors, a model of the interactions among nucleons is required; in the nonrelativistic limit one may use the nucleon number–density operator $\bar{n}n$. Using this, the mass form factor is

$$\begin{aligned} F(q) &= \int d^3x\, e^{-i\mathbf{q}\cdot\mathbf{x}} \frac{\rho_n(\mathbf{x})}{m_n} \qquad (43)\\ &\longrightarrow A \quad \text{when } q \to 0 \end{aligned}$$

Here $\rho_n(\mathbf{x})$ is the nuclear mass density. For typical heavy nuclei, the mass form factor can be approximated by the Helm form factor as

$$\begin{aligned} F(q) &= \frac{3j_1(qr_n)}{qr_n} e^{-(qs)^2/2} \qquad (44)\\ r_n &\approx 1.144 A^{1/3}\,\text{fm} \\ s &\approx 0.9\,\text{fm} \end{aligned}$$

First, let us compute the scattering rate for spin-independent scattering. From Eq. (37), performing the spin sum for spin-1/2 fermionic nuclei and DM yields $(\xi^\dagger_{t'}\xi_t) \to 1, \quad (\xi^\dagger_{s'}\xi_s) \to 1$, so that

$$|\mathcal{M}|^2 = \left(\frac{3C_V}{m_Z^2}\right)^2 (4m_\chi m_n)^2 \equiv b_n^2 (4m_\chi m_n)^2 \tag{45}$$

where b_n is the DM–nucleon vector coupling constant. Using Fermi's golden rule, the differential cross section is

$$d\sigma_n = \frac{|\mathcal{M}|^2}{4m_\chi m_n v} \frac{d^3p'}{(2\pi)^3 2m_\chi} \frac{d^3q}{(2\pi)^3 2m_n} (2\pi)^4 \delta^{(4)}(p+k-p'-k') \tag{46}$$

$$= \frac{b_n^2}{4\pi v^2} d|\mathbf{q}|^2\, d\cos\theta\, \delta\left(\cos\theta - \frac{|\mathbf{q}|}{2\mu_{\chi n} v}\right) \tag{47}$$

Here v is the incident DM speed, $\cos\theta = \hat{\mathbf{q}}\cdot\hat{\mathbf{p}}$, and $\mu_{\chi n}$ is the reduced mass of the DM–nucleon system. Integrating over all kinematically allowed $|\mathbf{q}| \le 2\mu_{\chi n} v$ yields the total cross section:

$$\sigma_n = \frac{\mu_{\chi n}^2}{\pi} b_n^2. \tag{48}$$

We now generalize the DM–nucleon scattering cross section to the DM–nucleus case. Rewriting the energy in terms of the nuclear recoil energy E_R and including the nuclear form factor, we obtain:

$$d\sigma_N = \frac{|\mathcal{M}|^2}{\pi} \frac{m_N}{2v^2} F^2(q)\, d\cos\theta\, \delta\left(\cos\theta - \frac{|\mathbf{q}|}{2\mu_{\chi N} v}\right) \tag{49}$$

$$\frac{d\sigma_N}{dE_R} = \frac{\sigma_n}{\mu_{\chi n}^2} \frac{m_N}{2v^2} F^2(q)\, \Theta\left(v - \sqrt{\frac{m_N E_R}{2\mu_{\chi N}^2}}\right) \tag{50}$$

In the second line, the squared matrix element is replaced by the single-nucleon cross section. The Θ-function enforces the minimum incident speed required to produce a recoil E_R.

The differential recoil rate per unit target mass and per unit time is defined by:

$$\frac{dR}{dE_R} = N_T\, n_\chi \int \frac{d\sigma}{dE_R} v\, f(\mathbf{v})\, d^3v \tag{51}$$

Here N_T is the number of target nuclei per unit mass, $n_\chi = \rho_\chi/m_\chi$ is the DM number density, and $f(\mathbf{v})$ from eq (14) is the laboratory-frame DM velocity distribution normalized to unity. To separate the velocity dependence, we define the mean inverse speed by selecting only DM with speed $v \ge v_{\rm min}$ from the speed distribution f($\mathbf{v}$) and taking the $1/v$-weighted average:

$$g(v_{\rm min}) = \int \frac{d^3v}{v} f(\mathbf{v})\, \Theta(v - v_{\rm min}), \tag{52}$$

$$v_{\rm min} = \sqrt{\frac{m_N E_R}{2\mu_{\chi N}^2}}. \tag{53}$$

Finally, allowing for different DM couplings to protons and neutrons, the recoil rate is rewritten as:

$$\frac{dR}{dE_R} = N_T \frac{\rho_\chi\, \sigma_n\, m_N}{m_\chi\, 2\mu_{\chi n}^2} \left(\frac{b_p Z + b_n(A-Z)}{b_n A}\right)^2 F^2(q)\, g(v_{\rm min}) \tag{54}$$

where b_n is the neutron coupling and b_p is the proton coupling.

The spin-dependent cross section takes a similar form:

$$\sigma_n = \frac{3\mu_{\chi n}^2}{\pi} \left(\sum_q \Delta_q^n \frac{C_A}{m_Z^2}\right)^2 \tag{55}$$

$$\equiv \frac{3\mu_{\chi n}^2}{\pi} \left(\sum_q \Delta_q^n a_q\right)^2 \tag{56}$$

For details on the spin-dependent cross section, see [6].

III. SUB-GEV DARK MATTER : DARK PHOTON MODEL

We have reviewed how WIMPs interact with nucleons. Since WIMPs were proposed, numerous direct-detection experiments have been conducted, but dark-matter signals have not been observed over large regions of the WIMP mass parameter. Our interest has therefore extended to lighter-mass dark matter, i.e., light dark matter (LDM).

From the previous lecture, the WIMP annihilation cross section is

$$\langle \sigma v \rangle \sim \frac{\alpha_\chi \alpha_f \, m_\chi^2}{m_V^4} \tag{57}$$

Therefore, when the mediator mass is $m_V \simeq 100\,\mathrm{GeV}$ and the dark-matter mass satisfies $m_\chi \lesssim \mathrm{GeV}$, $\langle \sigma v \rangle$ becomes too small and the dark-matter abundance becomes too large. This is the Lee–Weinberg bound, which constrains the dark-matter mass to be at least at the GeV scale.

A simple way to evade this is to consider DM with a very small electric charge that interacts via a photon-like mediator. Since $m_V \ll m_\chi$, the LW bound can be avoided, and the annihilation cross section is

$$\langle \sigma v \rangle \approx \frac{\pi \, \alpha_{\mathrm{em}}^2 \, Q^2}{m_\chi^2} \tag{58}$$

Taking the relic density into account, the DM mass is then given by

$$Q \simeq 10^{-3} \left(\frac{m_\chi}{\mathrm{GeV}} \right) \tag{59}$$

This is called the Dark Photon model.

Besides the Dark Photon model, there exist other LDM candidates such as axion-like particles (ALPs) and sterile neutrinos; in what follows we focus on the Dark Photon model.

A. Dark Photons

As discussed above, the Dark Photon model can evade the LW bound and does not require flavor considerations. Mixing can be used to address the relic abundance, and, moreover, it is an attractive dark-matter candidate in that it can be described by a simple $U(1)$ model.

In vacuum, the dark photon portal Lagrangian is

$$\begin{aligned} \mathcal{L} \supset & -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} \\ & + \frac{\varepsilon}{2} F_{\mu\nu} V^{\mu\nu} + \frac{1}{2} m_V^2 V_\mu V^\mu + e\, A_\mu J^\mu_{\mathrm{EM}} + g_\chi V_\mu J^\mu_D \end{aligned} \tag{60}$$

Here ε is the mixing parameter, J^μ_{EM} the electromagnetic current, and J^μ_D the dark current with gauge coupling g_χ. Throughout, we write the electric charge as $e = \sqrt{4\pi\alpha}$ with $\alpha \simeq 1/137$ the fine-structure constant.

First consider $m_V = 0$. With the field redefinition $\tilde{V}_\mu = V_\mu - \varepsilon A_\mu$, we obtain (see Appendix A for the calculation steps)

$$\begin{aligned} \mathcal{L} \supset & -\frac{1}{4}(1-\varepsilon^2) F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} \tilde{V}_{\mu\nu} \tilde{V}^{\mu\nu} \\ & + e\, A_\mu J^\mu_{\mathrm{EM}} + g_\chi (\tilde{V}_\mu + \varepsilon A_\mu) J^\mu_D \end{aligned} \tag{61}$$

Redefining $A_\mu \to A_\mu / \sqrt{1-\varepsilon^2}$ removes the coefficient of the photon kinetic term, and for $|\varepsilon| \ll 1$ the overall factor can be neglected.

The interaction terms are

$$\mathcal{L}_{\mathrm{int}} = e\, A_\mu J^\mu_{\mathrm{EM}} + g_\chi \tilde{V}_\mu J^\mu_D + \varepsilon g_\chi A_\mu J^\mu_D. \tag{62}$$

The photon field A_μ thus couples to both the SM current ($e\, A_\mu J^\mu_{\mathrm{EM}}$) and the DM current ($\varepsilon g_\chi A_\mu J^\mu_D$), whereas the dark vector $\tilde{V}_\mu$ couples only to the DM current.

Let the DM current be $J^\mu_D = \bar{\chi} \gamma^\mu \chi$. Then

$$\varepsilon g_\chi \, A_\mu J^\mu_D = e\, Q\, A_\mu \, \bar{\chi} \gamma^\mu \chi \qquad Q \equiv \frac{\varepsilon g_\chi}{e} \tag{63}$$

That is, the effective charge Q of DM is a millicharge; DM interacts strongly with the dark photon and only very weakly with the photon.

Now consider $m_V \neq 0$. Redefining $\tilde{A}_\mu = A_\mu - \varepsilon V_\mu$ and rearranging Eq. (60) similarly, the kinetic mixing term is removed and we obtain

$$\begin{aligned} \mathcal{L} \supset & -\tfrac{1}{4} \tilde{F}^2 - \tfrac{1}{4}(1-\varepsilon^2) V^2 \\ & + \tfrac{1}{2} m_V^2 V^2 + e\, \tilde{A}_\mu J^\mu_{\mathrm{EM}} + e\, \varepsilon \, V_\mu J^\mu_{\mathrm{EM}} + g_\chi V_\mu J^\mu_D \end{aligned} \tag{64}$$

Likewise, from the term $e\, \varepsilon\, V_\mu J^\mu_{\mathrm{EM}}$ one sees that the dark photon couples to the SM. In the $m_V \neq 0$ case, the constraints on the relic abundance differ for $m_V > m_\chi$ versus $m_V < m_\chi$; see [7, 8] for details.

IV. SUB-GeV DM DIRECT DETECTION : ELECTRON SCATTERING

Let us now discuss the generalized scattering framework for spin-independent dark matter, such as the Dark Photon.

Dark matter scatters off the target material, transferring energy and exciting the target from its ground state $|i\rangle$ to a final state $|f\rangle$. The incoming and outgoing dark-matter momentum eigenstates are $|p\rangle$ and $|p'\rangle$, respectively. As in the nuclear scattering case, we take

$$\mathbf{p} = m_\chi \mathbf{v} \tag{65}$$

$$\mathbf{p}' = \mathbf{p} - \mathbf{q} \tag{66}$$

where $\mathbf{q}$ is the momentum transferred to the target. The corresponding energy transfer is

$$\omega_q = \frac{1}{2} m_\chi v^2 - \frac{(m_\chi \mathbf{v} - \mathbf{q})^2}{2 m_\chi} = \mathbf{q} \cdot \mathbf{v} - \frac{q^2}{2 m_\chi} \tag{67}$$

This implies the kinematic limit

$$\omega_q \leq q v_{\max} - \frac{q^2}{2 m_\chi} \tag{68}$$

Using Fermi's golden rule, the scattering rate for a DM particle with velocity $\mathbf{v}$ is

$$\Gamma(\mathbf{v}) = \int \frac{d^3 q}{(2\pi)^3} \sum_f \left| \langle p', f \mid \delta \hat{H} \mid p, i \rangle \right|^2 2\pi \, \delta(E_f - E_i - \omega_q) \tag{69}$$

where $\delta\hat{H}$ is the interaction Hamiltonian. It is given by

$$\langle p'|\,\delta\hat{H}\,|p\rangle = \frac{1}{V}\int d^3x\, e^{i\mathbf{q}\cdot\mathbf{x}}\,\mathcal{V}(\mathbf{x}) = \frac{1}{V}\,\widetilde{\mathcal{V}}(-\mathbf{q}) \tag{70}$$

where V is the total spatial volume, $\mathcal{V}(\mathbf{x})$ is the effective scattering potential felt by the DM, and $\widetilde{\mathcal{V}}$ is its Fourier transform.

For spin-independent coupling, the potential is

$$\mathcal{V}(\mathbf{x}) = \int d^3x' \Big[n_p(\mathbf{x}')\,V_p(\mathbf{x}-\mathbf{x}') + n_n(\mathbf{x}')\,V_n(\mathbf{x}-\mathbf{x}') + n_e(\mathbf{x}')\,V_e(\mathbf{x}-\mathbf{x}')\Big], \tag{71}$$

where n_p, n_n, n_e are the proton, neutron, and electron number densities inside the target, and V_p, V_n, V_e are the corresponding scattering potentials generated by a single particle localized at the origin.

The Fourier transform then becomes

$$\widetilde{\mathcal{V}}(-\mathbf{q}) = \tilde{n}_p(-\mathbf{q})\,\tilde{V}_p(\mathbf{q}) + \tilde{n}_n(-\mathbf{q})\,\tilde{V}_n(\mathbf{q}) + \tilde{n}_e(-\mathbf{q})\,\tilde{V}_e(\mathbf{q}) \tag{72}$$

where $\tilde{n}_\psi$ and $\tilde{V}_\psi$ denote the Fourier transforms of n_ψ and V_ψ, respectively.

Let the in-medium couplings be f_p, f_n, f_e, and the corresponding vacuum values be f_p^0, f_n^0, f_e^0. We write

$$\tilde{V}_\psi(-\mathbf{q}) = \frac{f_\psi(\mathbf{q})}{f_\psi^0}\,\mathcal{M}_{\chi\psi}(q) \equiv f_\psi(\mathbf{q})\,\mathcal{M}_0(q), \tag{73}$$

where $\mathcal{M}_0(q)$ is the vacuum scattering matrix element for DM scattering off a constituent particle ψ (proton, neutron, or electron) with unit coupling. The total scattering potential becomes

$$\widetilde{\mathcal{V}}(-\mathbf{q}) = \Big[f_p(q)\,\tilde{n}_p(-\mathbf{q}) + f_n(q)\,\tilde{n}_n(-\mathbf{q}) + f_e(q)\,\tilde{n}_e(-\mathbf{q})\Big]\mathcal{M}_0(q) \tag{74}$$

This can be rewritten as

$$\widetilde{\mathcal{V}}(-\mathbf{q}) = \mathcal{M}_{\chi n}(q)\,\left[\frac{f_p(q)\,\tilde{n}_p(-\mathbf{q}) + f_n(q)\,\tilde{n}_n(-\mathbf{q}) + f_e(q)\,\tilde{n}_e(-\mathbf{q})}{f_n^0}\right] \tag{75}$$

$$\widetilde{\mathcal{V}}(-\mathbf{q}) = \mathcal{M}_{\chi e}(q)\,\left[\frac{f_p(q)\,\tilde{n}_p(-\mathbf{q}) + f_n(q)\,\tilde{n}_n(-\mathbf{q}) + f_e(q)\,\tilde{n}_e(-\mathbf{q})}{f_e^0}\right] \tag{76}$$

For convenience, we will henceforth denote $\mathcal{M}_{\chi n}$ or $\mathcal{M}_{\chi e}$ simply by $\mathcal{M}$, and define the target form factor $F_T(q)$ as the quantity in square brackets, representing the combined contributions from protons, neutrons, and electrons:

$$\widetilde{\mathcal{V}}(-\mathbf{q}) = \mathcal{M}(q)\,\mathcal{F}_T(\mathbf{q}) \tag{77}$$

The DM–target cross section is then written as

$$\bar{\sigma}_T \equiv \frac{\mu_{\chi T}^2}{\pi}\big|\mathcal{M}_{\chi T}(q_0)\big|^2, \qquad q_0 = m_\chi v_0, \tag{78}$$

where $\mu_{\chi T}$ is the DM–target reduced mass and v_0 is a reference velocity.

Since $\mathcal{F}_T$ is target specific, we define the dynamic structure factor as

$$S(\mathbf{q},\omega) \equiv \frac{1}{V}\sum_f \big|\langle f \mid \mathcal{F}_T(\mathbf{q}) \mid i\rangle\big|^2\, 2\pi\,\delta(E_f - E_i - \omega). \tag{79}$$

Substituting Eqs. (78) and (79) into (69), we obtain the generalized scattering rate for spin-independent scattering:

$$\Gamma(\mathbf{v}) = \frac{\pi\,\bar{\sigma}}{\mu^2}\int \frac{d^3q}{(2\pi)^3}\,\mathcal{F}_{\rm med}^2(q)\,S(\mathbf{q},\omega_q) \tag{80}$$

where $\bar{\sigma}, \mu$, denote $\bar{\sigma}_n, \mu_n$ or $\bar{\sigma}_e, \mu_e$.

Finally, the total rate is given by

$$R = \frac{1}{\rho_T}\,\frac{\rho_\chi}{m_\chi}\int d^3v\, f_\chi(\mathbf{v})\,\Gamma(\mathbf{v}) \tag{81}$$

A. Electron Transition

We now apply the above formalism to electronic transitions.

For electron energy eigenstates with creation operators $c_I^\dagger$, the initial state can be written as

$$|i\rangle = \prod_I c_I^\dagger |0\rangle, \tag{82}$$

and the final state as

$$|f\rangle = c_{I_2}^\dagger c_{I_1} |i\rangle, \tag{83}$$

which corresponds to a single electron being excited from an occupied state I_1 to an unoccupied state I_2.

The quantity $\mathcal{F}_T(\mathbf{q})$ can be written as

$$\mathcal{F}_T(\mathbf{q}) = \frac{f_e}{f_e^0}\, \tilde{n}_e(-\mathbf{q}) \tag{84}$$

$$= \frac{f_e}{f_e^0} \int \frac{d^3k'}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} (2\pi)^3 \delta^3(\mathbf{k}' - \mathbf{k} - \mathbf{q})\, \hat{c}_{\mathbf{k}'}^\dagger \hat{c}_{\mathbf{k}}. \tag{85}$$

The dynamic structure factor is then given by

$$S(\mathbf{q},\omega) = \frac{2\pi}{V} \left(\frac{f_e}{f_e^0}\right)^2 \sum_{I_1,I_2} \delta(E_{I_2} - E_{I_1} - \omega) \left| \int \frac{d^3k'}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} (2\pi)^3 \delta^3(\mathbf{k}' - \mathbf{k} - \mathbf{q})\, \langle i \mid \hat{c}_{I_1}^\dagger \hat{c}_{I_2} \hat{c}_{\mathbf{k}'}^\dagger \hat{c}_{\mathbf{k}} \mid i \rangle \right|^2 \tag{86}$$

$$= \frac{2\pi}{V} \left(\frac{f_e}{f_e^0}\right)^2 \sum_{I_1,I_2} \delta(E_{I_2} - E_{I_1} - \omega) \left| \int \frac{d^3k'}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} (2\pi)^3 \delta^3(\mathbf{k}' - \mathbf{k} - \mathbf{q}) \left\{\hat{c}_{\mathbf{k}}, \hat{c}_{I_1}^\dagger\right\} \left\{\hat{c}_{I_2}, \hat{c}_{\mathbf{k}'}^\dagger\right\} \right|^2. \tag{87}$$

The energy eigenstates can be expanded in momentum eigenstates as

$$c_I^\dagger |0\rangle = \int \frac{d^3k}{(2\pi)^3}\, \tilde{\psi}_I(\mathbf{k})\, c_{\mathbf{k}}^\dagger |0\rangle, \tag{88}$$

where $\tilde{\psi}_I(\mathbf{k})$ is the momentum-space wavefunction. Substituting this, we obtain

$$S(\mathbf{q},\omega) = \frac{2\pi}{V} \left(\frac{f_e}{f_e^0}\right)^2 \sum_{I_1,I_2} \delta(E_{I_2} - E_{I_1} - \omega) \left| \int \frac{d^3k'}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} (2\pi)^3 \delta^3(\mathbf{k}' - \mathbf{k} - \mathbf{q})\, \tilde{\psi}_{I_2}^*(\mathbf{k}')\, \tilde{\psi}_{I_1}(\mathbf{k}) \right|^2 \tag{89}$$

This expression can be applied to a wide range of target systems, including atoms, crystals, superconductors, and Dirac materials. We are now equipped to compute the rate for DM–electron scattering that induces electronic transitions.

To apply this framework to actual experiments, one must determine the dynamic structure factor using solid-state physics appropriate to each target material. Since light dark matter requires a highly precise theoretical description of the target, density functional theory (DFT) is commonly employed, and has been used to study materials such as Si, NaI, and graphene.

ACKNOWLEDGMENTS

Due to the time constraints, specific target materials being studied in Light Dark Matter Direct Detection have been omitted. Additionally, discussions on what results actual experiments should produce and detailed explanations of modulation have been left out. Topics such as phonons and magnons have also been excluded. If you have any questions about the missing parts, I will prepare them for the next lecture. I prepared this in a short time, so there are many insufficient parts and likely many errors in the lecture notes. I would appreciate your understanding.

Appendix A: Calculate the Dark Photon Portal

The definitions of $V_{\mu\nu}$ and $F_{\mu\nu}$ are as follows:

$$V_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu, \qquad F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{A1}$$

The redefined field is

$$\tilde{V}_\mu \equiv V_\mu - \varepsilon A_\mu \quad (\Rightarrow \ V_\mu = \tilde{V}_\mu + \varepsilon A_\mu). \tag{A2}$$

It follows that

$$V_{\mu\nu} = \partial_\mu(\tilde{V}_\nu + \varepsilon A_\nu) - \partial_\nu(\tilde{V}_\mu + \varepsilon A_\mu) = \tilde{V}_{\mu\nu} + \varepsilon F_{\mu\nu}. \tag{A3}$$

Substituting this, we have

$$\begin{aligned}
\mathcal{L}_{\text{kin+mix}} &= -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{4}V_{\mu\nu}V^{\mu\nu} + \tfrac{\varepsilon}{2}F_{\mu\nu}V^{\mu\nu} && \text{(A4)}\\
&= -\tfrac{1}{4}F^2 - \tfrac{1}{4}(\tilde{V} + \varepsilon F)^2 + \tfrac{\varepsilon}{2}F \cdot (\tilde{V} + \varepsilon F) && \text{(A5)}\\
&= -\tfrac{1}{4}F^2 - \tfrac{1}{4}\tilde{V}^2 - \tfrac{1}{2}\varepsilon\,\tilde{V}\cdot F - \tfrac{1}{4}\varepsilon^2F^2 + \tfrac{\varepsilon}{2}F\cdot\tilde{V} + \tfrac{\varepsilon^2}{2}F^2 && \text{(A6)}\\
&= -\tfrac{1}{4}(1-\varepsilon^2)F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{4}\tilde{V}_{\mu\nu}\tilde{V}^{\mu\nu}. && \text{(A7)}
\end{aligned}$$

[1] *The Hipparcos and Tycho Catalogues, Volume 1, Section 1.5: Transformation of Astrometric Data and Associated Error Propagation*, ESA (1997).
[2] E. D. N. M. Cirelli and P. Panci, JCAP 1310, 019 (2013), arXiv:1307.5955 [hep-ph].
[3] R. J. Hill and M. P. Solon, Phys. Rev. D91, 043505 (2015), arXiv:1409.8290 [hep-ph].
[4] B. G. F. Bishara, J. Brod and J. Zupan, JHEP 11, 059 (2017), arXiv:1707.06998 [hep-ph] ().
[5] B. G. F. Bishara, J. Brod and J. Zupan, arXiv:1708.02678 [hep-ph] ().
[6] C. K. P. Agrawal, Z. Chacko and . R. K. Mishra, arXiv:1003.1912 [hep-ph].
[7] H. L. C. Giovanetti, M. Lisanti and J. T. Ruderman, arXiv:2109.03246 [hep-ph].
[8] J. M.-R. L. J. Hall, K. Jedamzik and S. M. West, J. High Energy Phys. 03, 80 (2010), doi:10.1007/JHEP03(2010)080.